Review – Dark Matter (after Spring break)

Mass Estimators in Clusters of Galaxies

- virial $$M = \frac{2.5}{G}\sigma^2 R_e$$
- hydrostatic $$M = -\frac{r}{G}\frac{kT}{\mu m_p}\left(\frac{\partial \ln\rho}{\partial \ln r} + \frac{\partial \ln T}{\partial \ln r}\right)$$
- S.Z effect $$M \approx D_A^2 \frac{\int \Delta T \, d\Omega}{\langle T \rangle}$$
- weak gravitational lensing $$M(<\theta_I) = \left(1.1\times10^{14} M_\odot\right)\left(\frac{\theta_I}{30''}\right)^2\left(\frac{D_L}{D_S}\right)\left(\frac{D_{LS}}{1\,\mathrm{Gpc}}\right)$$

Gravitational Lensing

- Weak lensing: mild distortion of lensed image
- Strong lensing: strong distortion with multiple images
- Microlensing: temporary magnification from unresolved strong lensing

Strong & weak lensing divided by a critical surface density

$$\Sigma_{crit} = \frac{c^2}{4\pi G}\frac{D_S}{D_L D_{LS}}$$

Microlensing surveys now exclude macroscopic mass MACHO dark matter except for a narrow window of ~asteroid mass objects

3 Missing Mass problems

1. Cosmic DM $\Omega_m > \Omega_b$: must be non-baryonic
2. Cosmic missing baryon problem $\Sigma\Omega_i < \Omega_b$ SOLVED – in IGM
3. Local missing baryon problem

$$M_b < f_b M_{200} \text{ in individual objects}$$

Primordial Nucleosynthesis

BBN produces the light elements & their isotopes

$^1H \quad ^2H \quad ^3H \quad ^3He \quad ^4He \quad ^6Li \quad ^7Li$

The measured primordial abundances indicate $\Omega_b = 0.05$

This accounts for all the mass in STANDARD MODEL particles (mostly baryons = protons + neutrons in nuclei)

To get $\Omega_m \approx 0.3$, need new physics beyond the STANDARD MODEL

There are many paths to measure Ω_m.
A few include

- Cluster M/L : measure dynamical mass of clusters multiply resultin $\frac{M}{L}$ by j, the cosmic luminosity density
- Cluster baryon fractions : measure f_b & assume BBN so
$$\Omega_m = \frac{\Omega_b}{f_b}$$
- Weak lensing shear
- Peculiar velocities
$$\frac{\delta v}{v} \approx -\frac{1}{3}\frac{\Omega_m^{0.6}}{b}\frac{\delta \rho_g}{\rho_g}$$
- Power spectrum fits

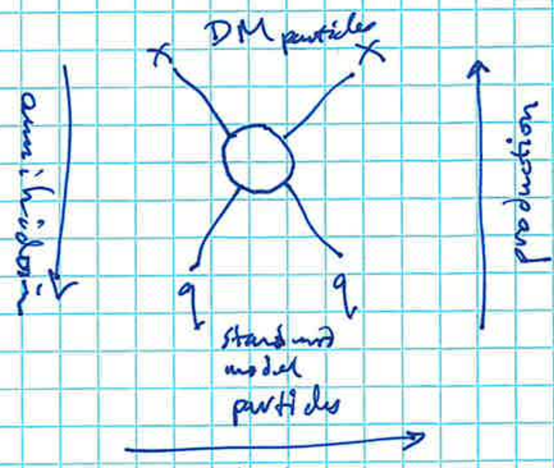
Galaxies : shape parameter $\Omega_m h \approx 0.21$

CMB fits : multiparameter fit BUT $\frac{\delta T}{T} \sim 10^{-5}$ not 10^{-3}

There are lots of others, but consistently $0.2 < \Omega_m < 0.33$

WIMP dark matter detection

The student should be able to describe experiments to detect WIMPs along each direction:



WIMPs are supersymmetric particles hypothesized to interact with standard model particles by way of the weak nuclear force

WIMP miracle: a weakly interacting particle naturally has the right relic (thermal) abundance to give $\Omega_\chi \approx 0.2$

Dark Matter candidates : non-standard model particles

COLD DM [WIMPs → WIMPzillas (very massive WIMPs) ; → light DM (below 2 GeV Lee-Weinberg limit)
Axions
etc.

Warm DM [sterile neutrinos
etc.

Hot DM [ordinary neutrinos
cannot solve the entire problem : $\Omega_\nu < \Omega_m$

Other DM [Self-interacting DM — requires new forces (dark photons) in the dark sector as well as new particles
Fuzzy DM
etc.

Hybrid DM [Dipolar DM — These attempt to reproduce MOND while remaining consistent with ΛCDM cosmology
Superfluid DM
Ghost condensate of AeST
etc.

Etc. [can always make up new invisible stuff

Most modified gravity theories can be excluded, but perhaps not MOND

MOND

rather than invoke DM, change force-law at an acceleration scale a_0

$a \gg a_0$: $a = g_N$ where g_N is the usual Newtonian force per unit mass: $g_N = \frac{GM}{r^2}$ for a point mass

$a \ll a_0$: $a \to \sqrt{a_0 g_N}$

In this deep MOND limit, the effective force is amplified by $\frac{a_0}{a}$

The Newtonian & deep MOND regimes are joined by an interpolation function that can be written either

$\mu(x)\, a = g_N$ where $x = a/a_0$ $\mu(x) \to x$ for $x \ll 1$

or

$a = \nu(y)\, g_N$ where $y = g_N/a_0$ $\nu(y) \to y^{-1/2}$ for $y \ll 1$

MOND predicts the five empirical laws of galactic rotation plus lots of other things, but leaves a residual missing baryon problem in clusters

External Field Effect in MOND

MOND violates the strong Equivalence Principle
as ~~location~~ local position invariance is violated:
the result of gravitational experiments (like dwarf galaxies)
depends on location: the same dwarf will have subtly
different kinematics if isolated or if near a giant host

Regime	Acceleration	Effective force	
Newtonian	$g_{int} >> a_0$	$a \rightarrow g_N$	
Still Newtonian	$g_{ext} > a_0 > g_{int}$	$a \rightarrow g_N$	Terrestrial lab
Quasi-Newtonian	$g_{int} < g_{ext} < a_0$	$a \rightarrow \left(\frac{a_0}{g_{ext}}\right) g_N$	
Deep MOND	$g_{ext} < g_{int} < a_0$	$a \rightarrow \sqrt{a_0 g_N}$	

MOND by itself is not a complete answer,

but its predictive successes imply that we're barking up
the wrong side of the dark matter tree.